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STRESS CONCENTRATION OF FINITE COMPOSITE LAMINATES WEAKENED BY MULTIPLE ELLIPTICAL HOLES

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Abstract—Based on the classical plate theory, a finite composite laminated plate weakened by multiple elliptical holes is treated as an anisotropic multiple-connected plate. Using the complex potential method in the plane theory of elasticity of an anisotropic body, a series solution to the title problem is obtained with the help of the Faber series expansion, the conformal mapping and the least squares boundary collocation techniques. The effects of the layups, the ellipticity of holes, the relative distance between holes, total number of holes and their locations on the stress concentration are studied in detail. Some useful conclusions are drawn.

INTRODUCTION

Composite materials, because of their high specific strength and stiffness and their flexible anisotropic property that can be tailored as per the requirements of applications, have found a variety of applications in many engineering fields, such as in aerospace, automobile and chemical engineering. As is known, holes and cutouts are inherent in many engineering structures, and cause serious stress concentration problems due to geometry discontinuity. These problems are even more serious in structures made of composite materials, since the materials exhibit anisotropic and brittle behavior. Therefore, much attention has been paid by many researchers to the stress concentration problems for composite structures. The closed form solution to stress concentration around a circular hole in an infinite orthotropic plate was first obtained by using the complex potential method (Lekhnitskii, 1957; Savin, 1961). Kosmodamianskii and Chernic (1981) obtained the stress states of an infinite plate weakened by two elliptical holes with parallel axes. Extensive studies have been made by the authors (Fan and Wu, 1988; Xu 1989, 1992; Yu and Fan, 1991, 1993a,b; Xu *et al.*, 1992) on the thermoelasticity problems of infinite laminated plates with multiple elliptical holes. Gerhardt (1984) obtained the solution to a finite plate weakened by a circular hole using the hybrid finite element method. Similar problems were studied by Ogonowski (1980), and Lin and Ko (1988) using the boundary collocation approach. These methods, however, still suffered from some drawbacks, such as large data preparations, long CPU time and low accuracy. According to the authors knowledge, there are no solutions to stress concentrations for a finite plate weakened by multiple elliptical holes in the literature. Thus, the objective of this paper is to obtain a solution to the title problem.

Based on the classical plate theory, the finite composite laminated plate with multiple elliptical holes is simplified as a multiple-connected homogeneous anisotropic plate. The series solution to the title problem is then obtained by using the complex potential method in the plane theory of elasticity of an anisotropic body with the help of the conformal mapping, the Faber series expansion and the least squares boundary collocation techniques. Furthermore, the effects of various parameters are discussed in detail, such as the relative distance between holes, the total number of holes, the locations of holes, and various loading conditions. Numerical results indicate that the present method yields accurate solutions, needs less CPU time and fewer data preparations over the existing methods, and is convenient to investigate effects of various parameters.

BASIC EQUATIONS

In the classical theory, the laminated composite plate is treated as an anisotropic plate. Therefore, the constitutive equation for a laminated plate in plane stress (Jones, 1975) is

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{16} \\ a_{12} & a_{22} & a_{26} \\ a_{16} & a_{26} & a_{66} \end{bmatrix} \begin{Bmatrix} \bar{\sigma}_x \\ \bar{\sigma}_y \\ \bar{\sigma}_{xy} \end{Bmatrix}, \tag{1}$$

where $\{\bar{\sigma}\}$ is the average in-plane stress and a_{ij} are the equivalent compliance coefficients depending on the fiber orientation and stacking sequence and on the property of each lamina.

In a rectangular coordinate system $x_i\,(i = 1,2,3)$, let u_i, σ_{ij} and ε_{ij} be the displacement, stress and strain, respectively. Suppose no body force exists, the basic equations of theory of an elastic body are

$$\begin{aligned} \varepsilon_{ij} &= \tfrac{1}{2}(u_{i,j}+u_{j,i}) \\ \sigma_{ij} &= E_{ijkl}\varepsilon_{kl} \\ \sigma_{ij,j} &= 0, \end{aligned} \tag{2}$$

where E_{ijkl} is elasticity coefficient, u_i, σ_{ij} and ε_{ij} are independent of the x_3 coordinate in the plane problem of elasticity theory. By introducing the Airy stress function, the solutions of eqn (2) can be expressed as (Lekhnitskii, 1957)

$$\begin{aligned} \sigma_x &= 2\,\mathrm{Re}\sum_{j=1}^{2} \mu_j^2 \varphi_j'(z_j) \\ \sigma_y &= 2\,\mathrm{Re}\sum_{j=1}^{2} \varphi_j'(z_j) \\ \tau_{xy} &= -2\,\mathrm{Re}\sum_{j=1}^{2} \mu_j \varphi(z_j) \end{aligned} \tag{3}$$

$$\begin{aligned} u &= 2\,\mathrm{Re}\sum_{j=1}^{2} p_j \varphi_j(z_j) - \omega y + u_0 \\ v &= 2\,\mathrm{Re}\sum_{j=1}^{2} q_j \varphi_j(z_j) + \omega x + v_0, \end{aligned} \tag{4}$$

where $\varphi_j(z_j)$ is an analytic function in the generalized region S_j by the affine transformation $z_j = x + \mu_j y$ from the physical region S, and $z_j = x + \mu_j y$; μ_j is the root of the characteristic equation [eqn (5)], a complex parameter representing the anisotropic extent of a laminated plate:

$$a_{11}\mu^4 - 2a_{16}\mu^3 + (2a_{12}+a_{66})\mu^2 - 2a_{26}\mu + a_{22} = 0, \tag{5}$$

where

$$\begin{aligned} p_j &= a_{11}\mu_j^2 + a_{12} - a_{16}\mu_j \\ q_j &= a_{12}\mu_j + \frac{a_{22}}{\mu_j} - a_{26}. \end{aligned} \tag{6}$$

When the forces X_n and Y_n are prescribed on the boundary, the boundary conditions are of the form

$$2\,\mathrm{Re}\sum_{j=1}^{2}\varphi_j(z_j) = \mp\int_0^t Y_n\,\mathrm{d}s + c_1$$

$$2\,\mathrm{Re}\sum_{j=1}^{2}\mu_j\varphi_j(z_j) = \pm\int_0^t X_n\,\mathrm{d}s + c_2. \tag{7}$$

The upper signs on the right-hand side of eqn (7) are taken for the outer contour, and the lower signs for the inner contour, i.e. for the contour of a cutout.

When the displacements u, v are prescribed on the boundary, the boundary conditions are of the form

$$2\,\mathrm{Re}\sum_{j=1}^{2}p_j\varphi_j(z_j) = u + \omega y - u_0$$

$$2\,\mathrm{Re}\sum_{j=1}^{2}q_j\varphi_j(z_j) = v - \omega x - v_0. \tag{8}$$

ANALYSIS

Consider a finite composite laminated plate weakened by multiple elliptical holes with contours L_0, L_1, $L_2, \ldots, L_l$, as shown in Fig. 1. Their semi-major, semi-minor axes and centers are a_m, b_m, and z_m ($m = 1, 2, \ldots l$), respectively. By affine transformation $z_j = x + \mu_j y$ from the region S onto the region S_j, the point z_m in the region S corresponds to the point z_{jm} in the region S_j.

Let the principal vector of forces acting on the contour of every hole be equal to zero; the complex potential function $\varphi_j(z_j)$ can be expressed as (Xu, 1992)

$$\varphi(z_j) = \varphi_{0j}(z_j) + \sum_{k=0}^{\infty} b_{jk}P_k(z_j), \quad (j = 1, 2), \tag{9}$$

where $\varphi_{0j}(z_j)$ is a holomorphic function in the infinite region with l elliptical holes. $P_k(z_j)$ is the Faber polynomial of the region limited by contour L_{j0}.

The mapping function is given as follows:

$$z_j - z_{jm} = R_{jm}\left(\xi_{jm} + \frac{t_{jm}}{\xi_{jm}}\right) \quad (m = 1, 2, \ldots l, \quad j = 1, 2), \tag{10}$$

where

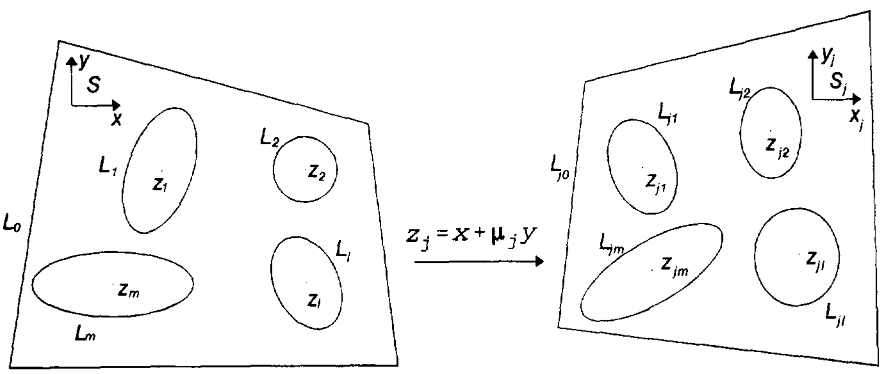


Fig. 1. The finite composite laminated plate weakened by multiple elliptical holes.

$$R_{jm} = \frac{a_m - i\mu_j b_m}{2}, \quad t_{jm} = \frac{a_m + i\mu_j b_m}{a_m - i\mu_j b_m}.$$

This mapping function transforms the exterior of hole m in the complex plane z_j into the exterior of a unit circle, $\xi_{jm} = \exp(i\theta)$, in complex plane ξ_{jm}. If the major and minor axes are not parallel to the coordinate axes, the rotation mapping should be resorted to.

Using Laurent series expansion and the Faber polynomial in the general region, the complex potential function can be shown as (Xu, 1989)

$$\varphi_j(z_j) = \sum_{m=1}^{l} \sum_{k=1}^{\infty} \frac{b_{jmk}}{\xi_{jm}^k} + \sum_{k=0}^{\infty} a_{jk} z_j^k. \tag{11}$$

Obviously, the complex potential function eqn (11) is analytic in the region S_j. Once the unknown coefficients b_{jmk} and a_{jk} are determined by using the boundary conditions, the stress and displacement field can be obtained uniquely.

Suppose the external forces X_n, Y_n are applied or the displacements $u(t)$, $v(t)$ are given on the contour, the boundary conditions eqns (7) and (8) can be expressed as

$$\sum_{j=1}^{2} [r_j \varphi_j(z_j) + s_j \overline{\varphi_j(z_j)}] = f(t), \tag{12}$$

where

$$r_j = 1 + i\mu_j, \quad s_j = 1 + i\overline{\mu_j}$$

$$f(t) = \pm \int_0^t i(X_n + iY_n)\,ds + c_1 + ic_2,$$

when the surface forces are given. The upper and lower signs correspond to the outer and inner contours, and

$$r_j = p_j + iq_j, \quad s_j = \overline{p_j} + i\overline{q_j}$$

$$f(t) = u(t) + iv(t) - i(v_0 + \omega x) - (u_0 - \omega y),$$

if the displacements are prescribed on the boundary.

The right hand side of eqn (12) can be expanded into the complex Fourier series, a power series of $\sigma = \exp(i\theta)$.

From the mapping function, eqn (10), it can be seen that function $\xi_{jm}(z_j)$ is holomorphic in the complex plane z_j weakened by the hole m. Therefore, function $\xi_{jm}^{-n}(z_j)$ is holomorphic in the interior of the p-th ($p \neq m$) hole and continuous to its boundary. Thus they can be expanded into a Faber series

$$\xi_{jm}^{-n} = \sum_{k=0}^{\infty} A_{n,k}^{jm} P_{kp}(z_j). \tag{13}$$

Similarly,

$$z_j^n = \sum_{k=0}^{\infty} H_{n,k}^{j} P_{kp}(z_j), \tag{14}$$

where $P_{kp}(z_j)$ is the k-th Faber polynomial for the ellipse L_{jp} of the complex z_j plane and

$$P_{kp}(z_j) = \xi_{jp}^k + \frac{t_{jp}^k}{\xi_{jp}^k}$$

$$P_{0p} = 1, \tag{15}$$

where the coefficients $A_{n,k}^{jm}, H_{n,k}^{j}$ in the Faber series can be determined by the Fourier expansion method (Xu, 1992). Substituting eqns (13) and (14) into the complex potential expression, eqn (11), and using $\xi_{jp} = \exp(i\theta) = \sigma$ in the contour L_{jp} of the p-th hole, the boundary values of $\varphi_j(z_j)$ ($j = 1,2$) are obtained in a power series of σ.

It is easy to prove (Xu, 1992) that the point $z = z_m + a_m \cos\theta + ib_m \sin\theta$ on the physical region is transformed into the point $\sigma = \exp(i\theta)$ on the ξ_{jm} plane by using the affine transformation $z_j = x + \mu_j y$ and the mapping transformation, eqn (10). Taking the values of $\varphi_j(z_j)$ a partial sum up to the N-th power term and substituting them into the boundary condition of every elliptical hole, and equating the coefficients of the same power σ^k ($k = 0, \pm 1, \pm 2, \ldots, \pm N$) on both sides of every equation, we obtain the $(2N+1)l$ linear equations about the coefficients b_{jmk}, a_{jk} and C_p ($p = 1,2,\ldots,l$), but these linear equations obtained from inner boundary conditions are not enough to determine all coefficients. Therefore, it is necessary to use outer boundary conditions. Although for smooth outer contour L_0 of the plate the accurate solution can be obtained using the Faber polynomial in the general region, the calculation is lengthy, complicated and untidy. A more convenient method, the least square boundary collocation technique, is used in this paper. Taking collocation points z_{ck} ($k = 1,2,\ldots, M, M \geqslant 2N+4$) along the outer contour L_0 and substituting z_{ck} into the boundary condition eqn (12), we can obtain the linear equations about the unknown coefficients b_{jmk} and a_{jk} that satisfy the outer boundary conditions. These equations together with $(2N+1)l$ equations that satisfy inner boundary conditions are used to determine the complex potential function $\varphi_j(z_j)$, and the stress field and displacement field in the laminated plate can be obtained by eqns (3) and (4).

Obviously, the complex potential function $\varphi_j(z_j)$ is an analytic function in the region S_j. Therefore, the accuracy of the solution can be judged according to whether the boundary conditions are satisfied fully. In the present method, the inner boundary conditions can be satisfied accurately (absolute error less than 10^{-5}). By increasing the number of collocation points, the outer boundary conditions can be better satisfied to ensure the relative error within one percent. As we know from the Saint–Venant principle, more accurate results (see Tables 1 and 2 for details) of the stress distribution around holes are obtained by using the present method.

NUMERICAL RESULTS

Consider a plywood plate in which the direction of the x-axis coincides with the maximum Young's modulus (Lekhnitskii, 1957) weakened by two circular holes with

Table 1. The stress σ_θ around hole 2 contour of the plywood plate with two holes subjected to $\sigma_x = 1.0$ by taking 32 collocation points on the outer boundary and the partial sum N

θ	$N = 1$	$N = 3$	$N = 5$	$N = 7$	$N = 9$	$N = 12$	$N = 15$	Lekhnitskii (1957)
0	−0.705	−0.706	−0.707	−0.707	−0.707	−0.707	−0.707	−0.71
15	−0.339	−0.339	−0.340	−0.340	−0.340	−0.340	−0.340	−0.34
30	0.069	0.069	0.069	0.069	0.069	0.069	0.069	0.07
45	0.403	0.403	0.403	0.403	0.403	0.403	0.403	0.40
60	0.962	0.962	0.963	0.963	0.963	0.963	0.963	0.96
75	2.567	2.568	2.569	2.569	2.569	2.569	2.569	2.57
90	5.450	5.452	5.454	5.454	5.455	5.455	5.455	5.45

Table 2. The stress σ_θ around hole 2 contour of the plywood plate with two holes subjected to $\sigma_x = 1.0$ by taking the partial sum $N = 3$ and $M = 4m$ collocation points on the outer boundary

θ	$m = 3$	$m = 5$	$m = 7$	$m = 9$	$m = 11$	Lekhnitskii (1957)
0	−0.70590	−0.70604	−0.70603	−0.70601	−0.70603	−0.71
15	−0.33919	−0.33929	−0.33927	−0.33926	−0.33927	−0.34
30	0.06904	0.06899	0.06900	0.06900	0.06900	0.07
45	0.40311	0.40308	0.40308	0.40309	0.40309	0.40
60	0.96244	0.96243	0.96244	0.96244	0.96244	0.96
75	2.56782	2.56783	2.56784	2.56784	2.56785	2.57
90	5.45174	5.45180	5.45181	5.45181	5.45185	5.45

diameter D, as shown in Fig. 2a. The material properties are

$$E_1 = 11.8\,\text{GPa} \quad E_2 = 5.89\,\text{GPa} \quad \nu_{12} = 0.071 \quad G_{12} = 0.687\,\text{GPa}.$$

Assuming that the relative center-to-center distance and the relative plate size are large enough ($l/D = 40$; $e/D = t/D = 40$), the numerical results for the finite plate with two holes

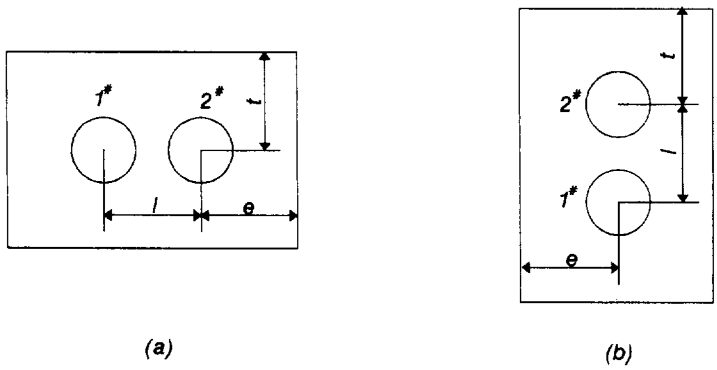


Fig. 2. The finite laminated plate weakened by two circular holes.

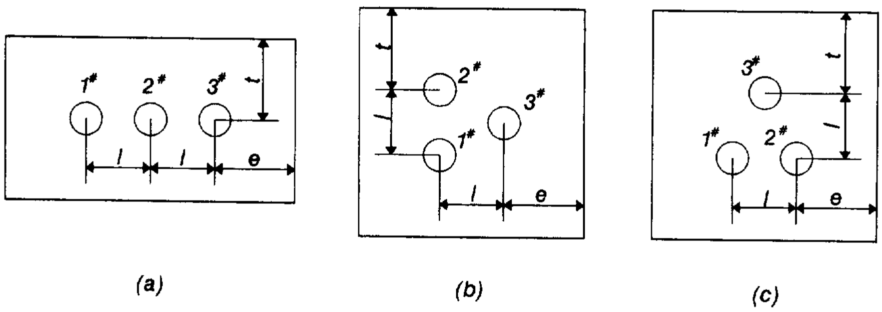


Fig. 3. The finite laminated plate weakened by three circular holes.

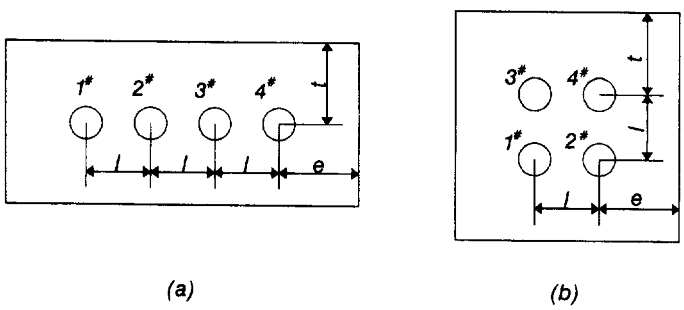


Fig. 4. The finite laminated plate weakened by four circular holes.

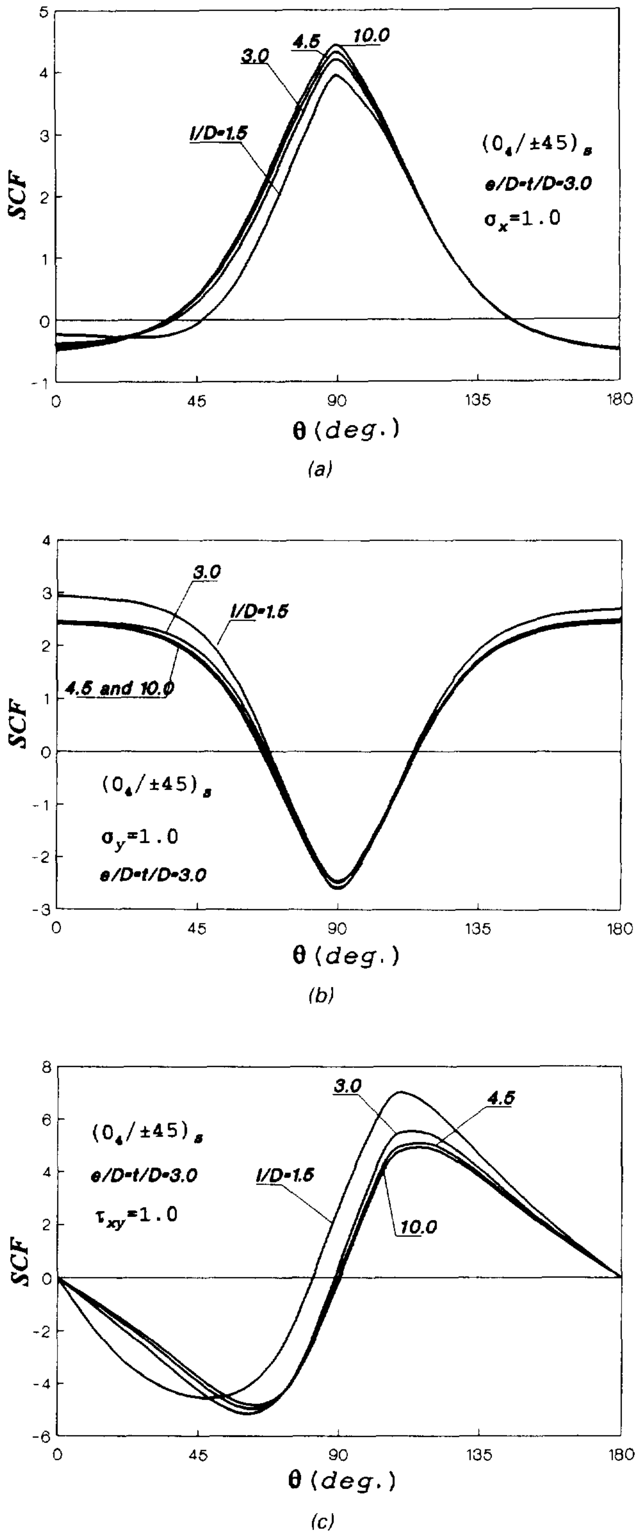


Fig. 5. The effect of the relative center-to-center distance on the stress concentration.

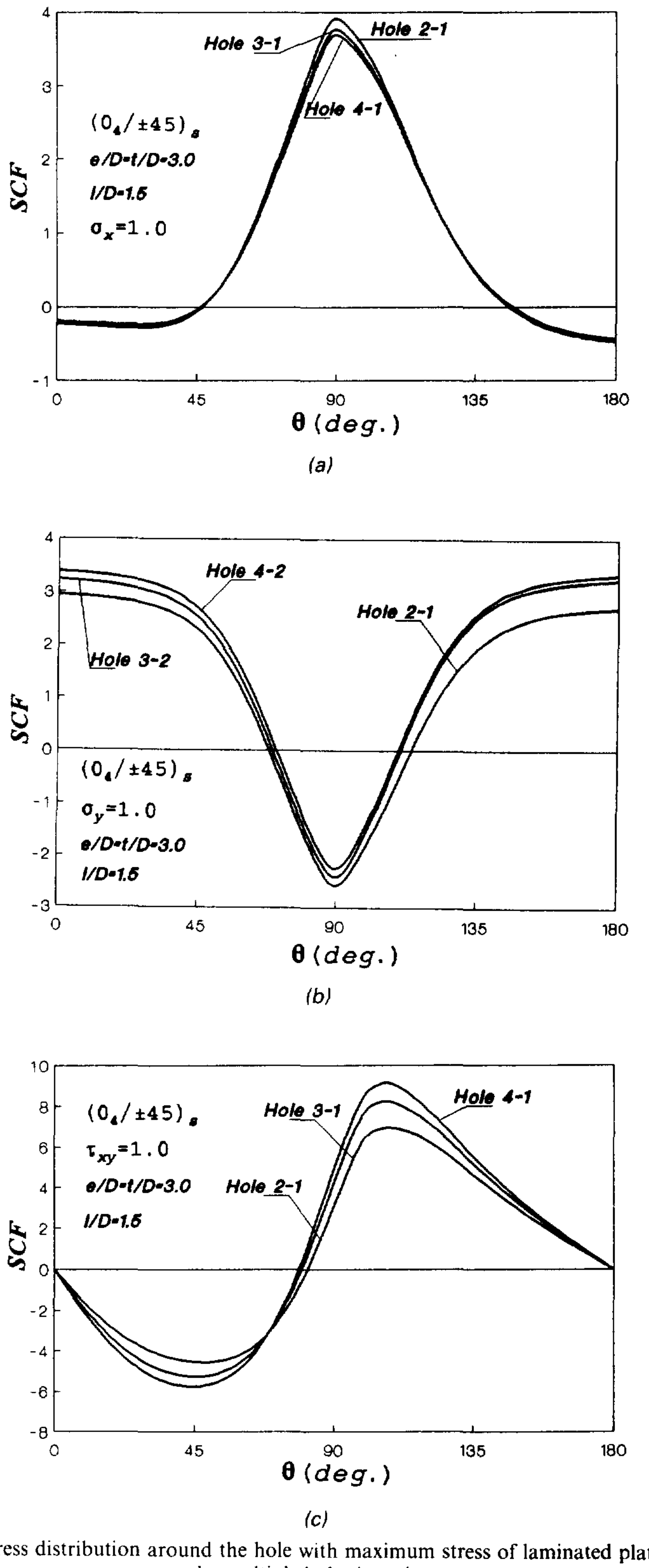


Fig. 6. The stress distribution around the hole with maximum stress of laminated plate weakened by multiple holes in series.

would be approximately equivalent to ones for the infinite plate with a single hole, which were calculated by Lekhnitskii (1957). The results by the present method by taking the partial sum N and $M = 4m$ collocation points on the outer boundary are compared with Lekhnitskii's results in Tables 1 and 2, where m is the number of collocation points uniformly distributed on each side. It is shown that the present solution has the features of fast convergence and high accuracy.

The numerical examples presented in this paper have focused on the stress concentration of composite laminated plates with multiple holes. The effects of many parameters such as the size of laminated plate, the layups and the hole eccentricity, etc., on the stress concentration of a plate with a single hole have been discussed elsewhere by the present author (Xu, 1992). Those effects are the same for a plate weakened by multiple elliptical holes. In the following calculation, the partial sum N and the number of collocation points on the outer boundary are taken to be 10 and 32, respectively. Numerical results indicate that the accuracy to satisfy boundary conditions has exceeded the requirement mentioned previously.

Consider a finite laminate plate $(O_4/\pm 45)_s$ weakened by multiple circular holes with diameter D, as shown in Figs 2–4. The laminated plate is composed of T300/5222. The material properties are

$$E_1 = 126.2\,\text{GPa} \quad E_2 = 7.3\,\text{GPa} \quad \nu_{12} = 0.247 \quad G_{12} = 4.5\,\text{GPa}.$$

Assume that these plate are, respectively, subjected to σ_x, σ_y and τ_{xy} along the outer boundary. The stress concentration factor is SCF $= \sigma_\theta/\sigma$, and the minimum distance from the center of the hole to two sides of the plate is $e = 3D$, $t = 3D$, respectively.

Figure 5(a–c) shows the stress σ_θ distribution around hole 1 of the plate with two circular holes shown in Fig. 2(a). The stress distribution around hole 2 can be obtained by means of the reflection principle. The effect of the relative center-to-center distance l/D on the stress concentration is easily seen. When the relative center-to-center distance l/D becomes smaller, the stress concentration decreases rapidly for the plate under the σ_x loading, and conversely the stress concentration increases rapidly for the plate acted by σ_y and τ_{xy}. When $l/D \geqslant 4.5$, the effect of the relative center-to-center distance on the stress distribution becomes very small, and the plate with multiple holes can be treated as with a single hole in engineering analysis. It should be pointed out that differential loads acting on the plate cause different effects of the relative distance l/D on the stress concentration, because the sudden change of stiffness caused by geometric discontinuity by a series of holes becomes less evident. Thus, it is beneficial for the reduction of the stress concentration.

Figure 6(a–c) shows the stress distribution around the hole with maximum stress in the laminated plate containing multiple holes in series (Figs 2a, 3a and 4a) under loads of

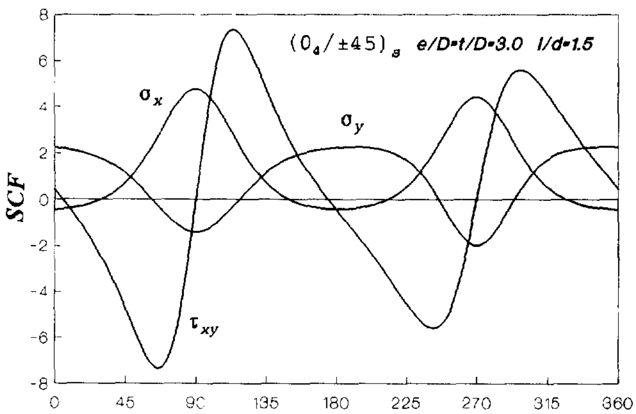


Fig. 7. The stress σ_θ pattern around hole 1 for the plate with two holes shown in Fig. 2(b).

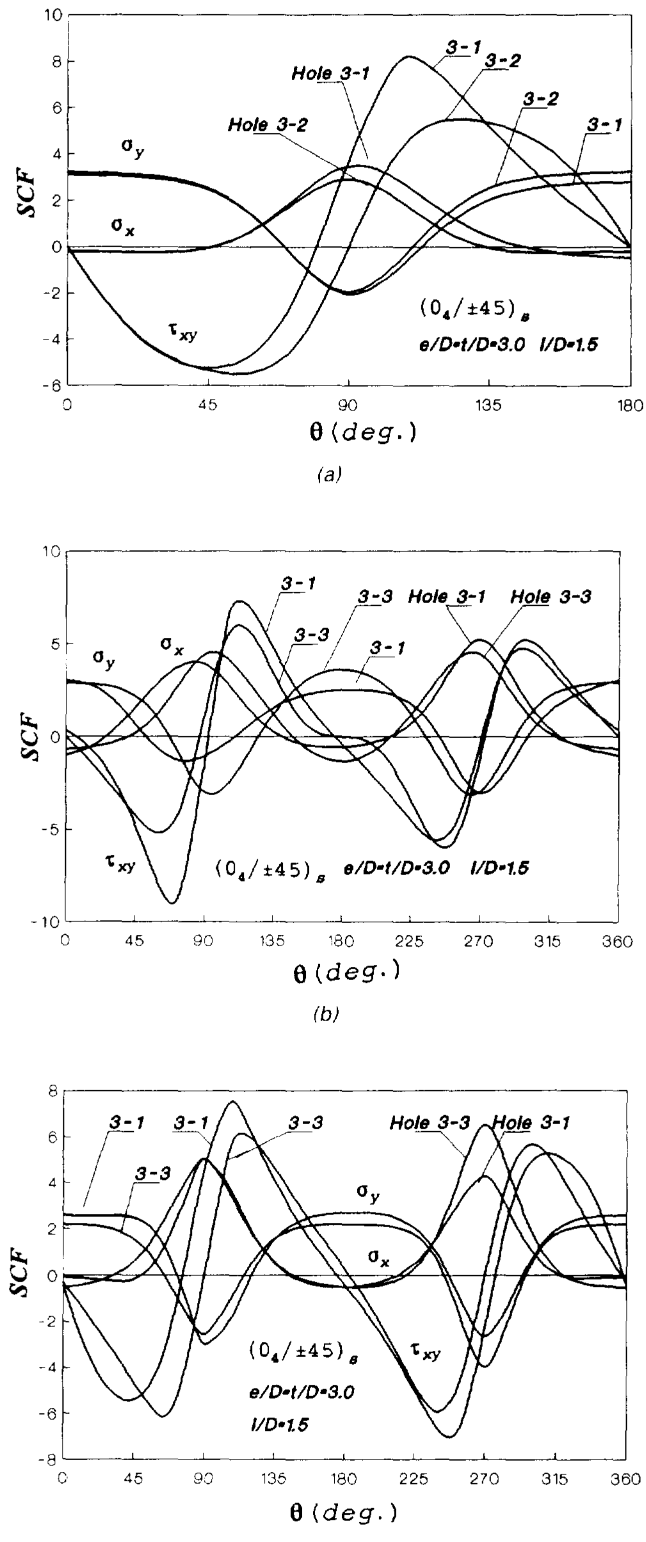
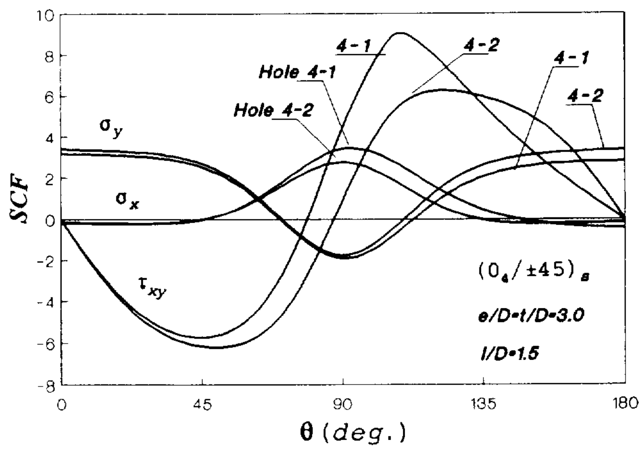


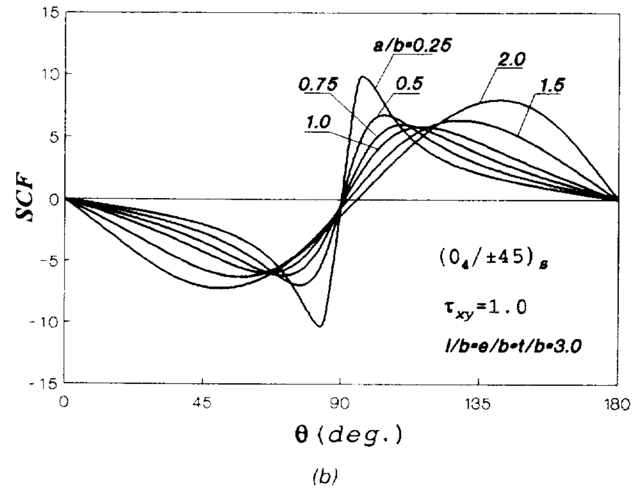
Fig. 8. The stress σ_θ pattern around the hole with maximum stress of laminated plate weakened by three holes shown in Fig. 3.

σ_x, σ_y and τ_{xy}, respectively. The results indicate that when σ_x is applied, the increase of number of holes decreases stress concentration for the series of holes arrangement, and when σ_y or τ_{xy} is applied on the plate, the increase of number of holes causes a more severe stress concentration. The reason for this phenomenon is the same as that for a plate under different loads. Owing to the interaction among holes, the maximum stress concentration is produced in two side holes and the middle hole, when σ_x (or τ_{xy}) and σ_y are applied in the plate, respectively.

The following discussion focuses on the effect of the hole position changes on the stress concentration. Figure 7 describes the stress σ_θ pattern around hole 1 for the plate with two holes subjected to σ_x, σ_y and τ_{xy}, respectively (Fig. 2b). Compared with the solution (Fig. 5) of the plate with two holes shown in Fig. 2(a), the stress concentration of holes shown in Fig. 2(b) is more severe when σ_x and τ_{xy} is applied, but decreases stress concentration for case Fig. 2(b) when σ_y is applied. Figure 8(a–c) presents, respectively, the stress distributions around the hole with maximum stress for the plate weakened by three holes, as shown in Fig. 3(a–c). The numerical results show that the stress concentration is the most severe for case (b), and that the plate shown in Fig. 3(a) has the least stress concentration around holes when σ_x is applied. Under loading σ_y, the most severe case is case (c) and case (a) has the least stress concentration. When τ_{xy} is applied, the most severe case is (b) and case (c) has the smallest stress concentration. Figure 9(a and b) shows the stress around the hole with maximum stress for the plate containing four holes as shown in Fig. 4(a and b). Cases



(a)



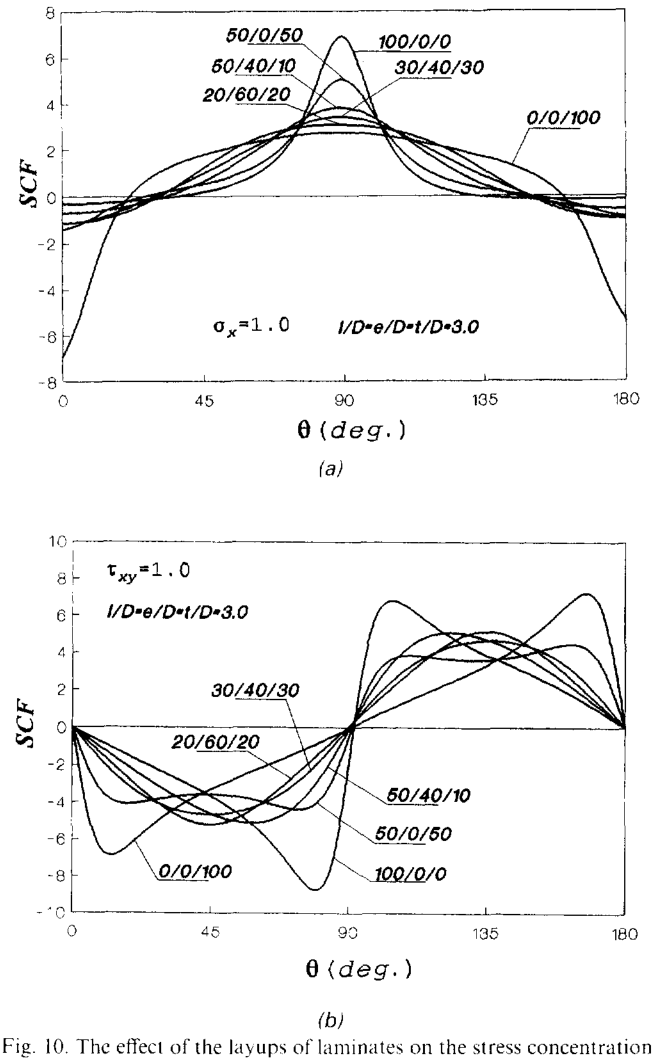
(b)

Fig. 9. The stress σ_θ pattern around the hole with maximum stress of laminated plate weakened by four holes shown in Fig. 4.

(b), (a) and (c) have the most severe stress concentrations under loading of σ_x, σ_y or τ_{xy}, respectively. From the above discussion, one can conclude that placing holes to cause smooth changes of stiffness along the loading direction is beneficial for the decrease of the stress concentration. All results indicate that when the plate is subjected to shear stress, the stress concentration is the most serious, the same as in the case of isotropic plates.

Figure 10 shows the stress distributions around the hole 1 of the laminated plate $(0_x/\pm45_\beta/90_\gamma)_s$ with two holes (Fig. 2a). The results show that the stress concentration strongly depends on the percentage of each layup in the whole plate. The more anisotropic the plate, the more severe the stress concentration. The increase in the number of $\pm45°$ laminae causes a decrease in stress concentration because it reduces the extent of the anisotropy of laminates.

Figure 11 show the stress concentrations around the hole 1 of the laminated plate $(0_4/\pm45)_s$ weakened by two elliptical holes (Fig. 12). In general, the increase of the ellipticity causes more severe stress concentrations. It is interesting, however, that when $a/b > 1.0$, the stress concentration becomes even smaller than that of a circular hole ($a/b = 1.0$) in the plate subjected to σ_y. For a specific laminated plate subjected to a specific load, there exists a hole of specific ellipticity which causes the smallest stress concentration; in general, it is not a circular hole. The value of a/b strongly depends on the anisotropic behavior of the plate and its loading conditions.



(b)

Fig. 10. The effect of the layups of laminates on the stress concentration.

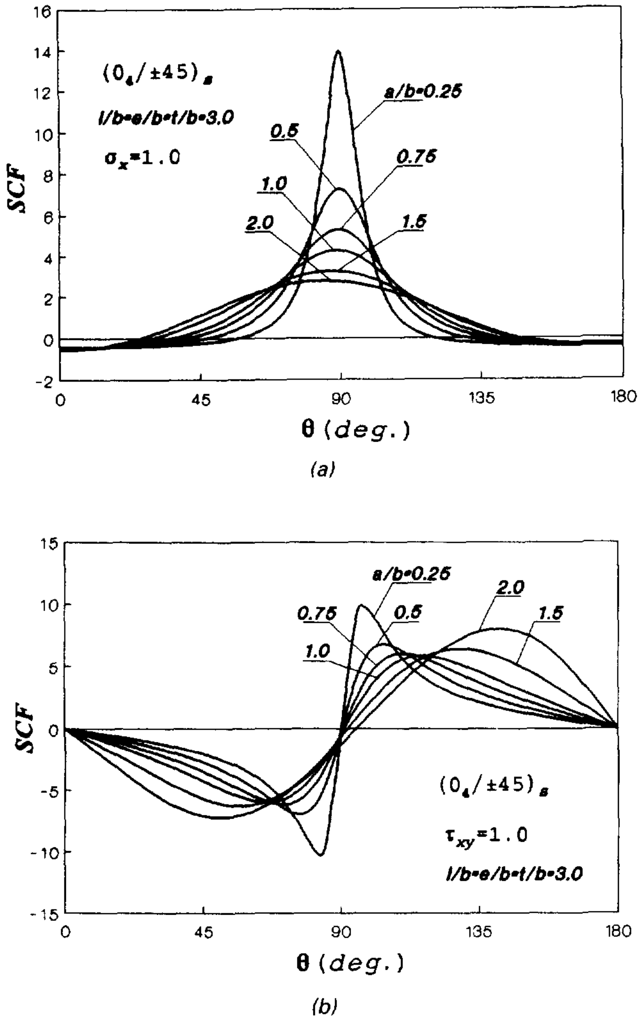


Fig. 11. The effect of the ellipticity on the stress concentration.

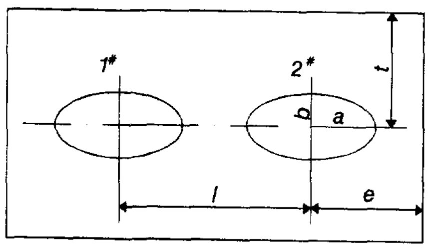


Fig. 12. The finite laminated plate weakened by two elliptical holes.

CONCLUSIONS

Based on the results reported in this paper, the following conclusions may be drawn. (i) The effect of the relative center-to-center distance, l/D, on the stress concentration is obvious. In general, the shorter the relative distance, the larger the stress concentration, but when the direction of the series of holes is the same as that of the applied force, the phenomenon is opposite. (ii) The increase of number of holes along the direction of applied force is beneficial for the decrease of the stress concentration, but in all other cases, the increase of number of holes causes more severe stress concentration. (iii) The effect of the arrangement of holes on the stress concentration is obvious. The arrangement which causes the smooth stiffness changes along the direction of acting force is beneficial to the decrease of the stress concentration. (iv) The stress concentration strongly depends on the layups of laminates. The increase of the number of $\pm 45^{\circ}$ laminae is beneficial to the decrease of stress concentration because it reduces the extent of the anisotropy of laminates. (v) In general, with the increase of ellipticity, the stress concentration becomes more serious. However, for a specific laminated plate under a specific loading there exists a hole of specific ellipticity which causes the smallest stress concentration, and in general, it is not a circular hole.

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